AMERICAN JOURNAL of PHYSICS

Secondary School Mathematics in Relation to College Physics

Paul S. Epstein

Citation: American Journal of Physics **9**, 34 (1941); doi: 10.1119/1.1991626
View online: http://dx.doi.org/10.1119/1.1991626
View Table of Contents: http://scitation.aip.org/content/aapt/journal/ajp/9/1?ver=pdfcov
Published by the American Association of Physics Teachers

Secondary School Mathematics in Relation to College Physics

Paul S. Epstein
California Institute of Technology, Pasadena, California

THE problems of mathematical instruction in elementary and secondary schools are of great professional importance to the physics teacher. In particular, the physics instructor of collegiate standing is much interested in the following question: "*Is the mathematical equipment which the college freshman brings from the secondary school all that his teacher in physics would like it to be?*" Specific as this question is, it is an organic part of the wider problem of secondary and elementary education in general and cannot be treated aside from the broader educational trends prevailing in our schools. In view of the complexity of the subject, the present paper does not claim to offer an exhaustive analysis but merely attempts to serve as a starting point for its further discussion.

It is a well-known fact that the dominant point of view in contemporary education is that of the so-called practical or pragmatic philosophy. Seen from this vantage point, the purpose of the schools is to help the students in the absorption and interpretation of experience. Many educators maintain, therefore, that knowledge should not be imparted in abstract form but only through its practical applications. It must be admitted that it is sound educational policy to point out, wherever possible, the practical utility of the teaching material. It is well to show the students that the subject matter is not cut and dried but alive and valuable insofar as it is helpful in dealing with the everyday problems even of their limited world. This goes a long way toward keeping up their interest and making the process of learning easy and pleasant. However, the modern educator should beware of a one-sided exaggeration of the pragmatic approach to the exclusion of every other method of instruction. The danger of such an overestimation is heightened by the name *progressive education*, which has been given to the several systems of teaching that try to carry pragmatism into effect—a name which suggests that these trends are entirely new and modern. Certain secondary features of the progressive systems are really novel and highly commendable in their tendencies: in the relations of the pupils with their teacher, they are intended to moderate discipline by comradeship; in the relations of the pupils with one another, to replace the competitive spirit by the spirit of teamwork. These are praiseworthy endeavors, although the success of the present schemes for their realization is not yet beyond question. However, the primary essence of the so-called progressive education, *its utilitarian character and insistence on practical application*, is neither new nor modern. William James, the founder of pragmatic philosophy, described it as "a new name for old ways of thinking." This description is particularly apt in relation to pragmatic education whose roots lie in the pioneer psychology of the last century. In the face of a vast, untamed continent the emphasis of the pioneers lay naturally on action. Lack of time and personnel in the little red schoolhouses were compelling reasons for sacrificing all long-range aims of instruction to its immediate utility.

Having given praise to the advantages of progressive education, we must also mention the drawbacks which may manifest themselves in elementary and secondary mathematical instruction. The extreme stress on teaching through applications may lead to some undesirable features, which we shall now discuss. In the first place, the emphasis lies on the practical side and not on the underlying principles, on *how to do things and not how to reason them out.* Altogether too often we meet teachers who—without fully explaining its derivation—merely give the students a formula for the solution of a certain type of problems and assign to them 40 or 50 problems for exercise. Attempts by the pupils to solve these problems in their own way are not only discouraged but strictly forbidden. Teachers of this sort have, in our opinion, a completely wrong view of the function of mathematics in education. Indeed, we may ask: "Is the knowledge of how to give change, how to compute percentages or how to calculate the heights of mountains a sufficient compensation for devoting to mathematics one's adolescent years?" We think this is open to serious doubt. Mathematics is a valuable part of the curriculum because it offers a wonderful opportunity for developing the powers of logical reasoning—powers which once acquired remain, when many of the rules of arithmetic and all the theorems of geometry are long forgotten.

The reader will have noticed that we are touching here upon the so-called problem of *transference of training*, the question whether practice in one kind of mental operations is of any help in acquiring the mastery of another kind. The experienced educators have always claimed that the answer to this question is in the affirmative, that the popular phrase of "improving the mind" has a basis in fact, and that it should be the highest aim of general education. For a few years, this contention was questioned by psychologists whose findings seemed to show little or no transference. However, as the conditions of the psychological experiment were diversified, it was found that the negative result of the early investigations was due to a choice of experimental conditions having but little analogy to schoolroom situations. Experiments designed in sufficient conformity to schoolroom practice to have a bearing on education gave an entirely different result. One of the most careful and comprehensive investigations in this connection is due to H. Woodrow,[1] who states his conclusions in the following words:

> The experiment shows that in a case where one kind of training—undirected drill—produces amounts of transference which are sometimes positive and sometimes negative, but always small, another kind of training with the same drill material may result in a transference, the effects of which are uniformly large and positive . . . the difference between unenlightened drill and intelligent teaching.

This is exactly the point that we are trying to make: mechanical application of rules and formulas is worthless for the development of the reasoning powers, but a systematic instruction in mathematics may be of great benefit for it.

This great opportunity is thrown away by those teachers who conduct their classes in the mechanical parrot-like way described above. But this is not all: *the utilitarian adjustment of the instruction to special problems defeats its own purpose.* When the type of problem is but slightly changed, the pupil is completely helpless. As the little daughter of some friends of mine remarked to me: "Oh yes, I know arithmetic very well. I know how to divide and how to multiply. The only thing I don't know is when to divide and when to multiply." It is remarkable that people in practical life engaged in training for gainful professions do not believe in teaching through applications only. The singing teacher, the athletic coach make their students go through long and arduous preliminary work before permitting them to engage in practice. A few years ago I happened to visit the scientific laboratories of a large industrial plant and was struck by the apparently nonutilitarian character of the research work there in progress. Being introduced to one of the heads of the firm, a vice president in charge of sales, I complimented him on the excellence of the laboratories and mentioned that I saw there only pure science, the same kind that we carry on in universities. The businessman smiled and answered:

> It turned out to be the best policy to let the scientists work in their own way. They dig at the roots of the phenomena. When they understand the fundamental principles involved, it is for them child's play to deal with any practical applications that may come up. The scientific laboratories give us not only prestige but profit; they have paid for themselves scores of times over.

Another undesirable feature of teaching through applications lies in the fact that information is imparted *as opportunity arises*—often in a haphazard way. Unless the teacher is exceptionally skillful, the instruction remains one-sided and incomplete in that the different parts of the subject matter seem to stand by themselves and their interdependence is not brought out. The subject matter appears to the student as a series of disconnected rules and not as a unified science. In this unsystematic procedure some topics are apt to be left out altogether, especially those that seem to the pupils difficult or uninteresting. We have already mentioned that it is well to make the instruction as easy and interesting as possible. This principle is now recognized so universally that we are apt to forget that in former times it was in question. A hundred years ago the English philosopher James Mill ridiculed

[1] H. Woodrow, J. Ed. Psy. **18**, 159 (1927).

endeavors to make instruction easier with the words: "Next, they will be inventing schemes to make prayer easier." Fortunately the times are past when education was considered exclusively from the point of view of discipline, and the mental comforts of the pupils were entirely disregarded. In mathematics the efforts to enlist the interest of the students have begun earlier than in other subjects, but there is still room for improvement. For instance, many of the problems could be taken from the hobbies and activities of the adolescent and so be brought nearer to his sphere of interests.

However, trying to make a subject palatable to the pupil by sparing him boredom from unnecessary abstractions is one thing; and cutting out whole parts of the curriculum which seem unattractive, although they may be indispensable for a complete understanding of the whole, is another. Such a policy is short sighted because it protects the pupil from slight mental discomforts in the beginning at the expense of more formidable difficulties in the end. The interest of the adolescent is mightily stirred when the different bits he has learned begin to fall into place and mathematics reveals itself to his maturing eye as a united and harmonious structure. On the other hand, his interest must necessarily flag if such an integration of his knowledge is made impossible and what should be an edifice remains to him a pile of bricks.

I have emphasized the evils of a one-sided exaggeration of pragmatism for the reason stated in the beginning: because this philosophy is part and parcel of our contemporary school policies. In fairness to the secondary school teachers I must say, however, that the extreme cases described by me are apt to occur far oftener in elementary than in secondary schools. The tendencies of progressive education affect the teaching of geometry, algebra and trigonometry in a lesser degree; but if their influence is here weaker, it is by no means negligible. Thus the preceding discussion has a twofold bearing on the specific question in which we are most interested—the adequacy of secondary school teaching for college needs. In the first place, arithmetic is the foundation of all mathematical teaching. Therefore, faulty instruction in arithmetic in the lower grades may impede the effective teaching of more advanced mathematics in the higher. The harm done in the elementary school is but slowly and painfully repaired in the secondary. If it is undone incompletely, it comes to the attention of the college instructor. In the second place, the lack of balance between applications and fundamentals occurs in the teaching of arithmetic in its most virulent manifestations described above. Here the virus can be studied, so to speak, in its pure culture; but the same problem presents itself often in a milder and less obvious form in the teaching of the higher branches of mathematics. In this case the conditions are even more complicated because not only two but fully three aspects of the subject vie for the attention of the teacher. The three-cornered rivalry lies between, first, the practical and numerical applications and examples; second, the formalistic drill or so-called mathematical technic; third, the conceptual foundations and basic logical connections. The question is: Do all three aspects receive sufficient attention, and how much time should be devoted to each?

The pragmatic tendencies of our time favor the numerical applications which bring in relief the utility of mathematics. Therefore, the treatment of this aspect is everywhere adequate or more than adequate. The secondary educators have become aware also of the broadening influence of the conceptual foundations and of that class of problems which are not so much concerned with the mechanical application of a formula as with reasoning out which formula to apply. Some of the larger high schools are conducting special courses for more gifted students in which the logical connections are particularly stressed. Both these phases of mathematical instruction are of general educational value: the second, *directly* because it sharpens the mind and develops the powers of logical reasoning; the first, *in a subsidiary way*, as it helps to maintain the student's interest and gives him a feeling of accomplishment.

It is different with the third aspect, *the mathematical technic*, meaning by this the skill in the transformations, reductions and simplifications of mathematical expressions. Only the student who intends to go into higher mathematics, physics or engineering needs this skill, but then he needs it badly. It has not much general educational value but it happens to be the prerequisite in which the collegiate teachers in physics and calculus are most interested. This is another example of the naive and oversimplified way in which the progressive theories of teaching through experience approach highly complicated questions; precisely the students of engineering who want to use mathematics as the tool of their profession must be versed not so much in the numerical applications of algebra and trigonometry as in something that appears to the outsider to be an idle intellectual game.

The only complaint about secondary school mathematics which I have heard in my many conversations with freshman instructors is of an insufficient training in mathematical technic. In all other respects they find the preparation of the students good or, at least, adequate. The deficiency in mathematical skill manifests itself particularly in trigonometry. The high school exercises are devoted mostly to the solution of triangles and similar numerical problems, while from the point of view of the physicist training in transformations of formulas would be preferable, supplemented by problems involving the interrelations of the different trigonometric functions, their addition theorems, and other properties. In a lesser degree similar considerations refer to algebra. A little more experience in the manipulation and transformation of algebraic expressions would give a great advantage to the college freshman bent on the study of calculus.

In a most gratifying spirit of cooperation mathematics departments of the high schools in our locality begin to meet these desires. The Pasadena Senior High School is offering now two types of courses in trigonometry. The one emphasizes the calculation of triangles and is primarily designed for those students who prepare for surveying; the other stresses the trigonometric functions and is meant for those who intend to go into physics and engineering. This division solves the problem in a most satisfactory way; its general adoption by the larger high schools would, certainly, find the cordial support of the collegiate instructors. The situation is more difficult in smaller schools which have not enough personnel to offer two courses. The majority of the students are not preparing themselves for the study of physics or engineering and they have no use for the tricks of mathematical sleight-of-hand. We have already expressed the view that the primary function of school mathematics is to teach the students how to use their reasoning abilities, and we have seen that this view is supported by the findings of modern experimental psychology. If it be accepted, then it is possible to argue that the students of average mathematical ability, who constitute the majority, are benefited by a thorough instruction in mathematics fully as much as the few gifted ones, provided the instruction emphasizes fundamentals of logic rather than mathematical technic. The curriculum, of course, should be adjusted to the needs of the majority, but there is no reason why students who are willing to go beyond the requirements of the course and who wish to exercise their superior gifts by working special problems should not be allowed and encouraged to do so. The special problems assigned to them should be of the kind developing mathematical technic.

The question may be raised as to how the college teachers of physics could make their influence felt. The first step toward this would be to make their desires—with respect to the secondary teaching of mathematics—articulate and precise. The American Association of Physics Teachers is aware of the importance of the problem and has devoted at least one of its sectional meetings, on the Pacific Coast, to a discussion of it. The practice of publicly threshing it out in meetings and in periodicals should be continued and supplemented by the appointment of standing committees charged with drawing up a detailed list of desiderata. There will be no difficulty in obtaining the cooperation of the secondary schools. The relations of the California Institute of Technology with the mathematics departments of the neighboring secondary institutions always have been most pleasant; any suggestions with respect to mutual needs have been treated with sympathy and understanding. But, when the collegiate physics teachers have a concrete program backed by the majority of the profession, they will be able to obtain the consideration of their wishes even in elementary schools.